Solutions 1a

Problem 1: Sum and product notation. Wrtie out the following sums and products and simplify when possible.

1. $\sum_{k=1}^{4}(k-1)^2 = (1-1)^2 + (2-1)^2 + (3-1)^2 + (4-1)^2 = 0 + 1 + 4 + 9 = 14$
2. $\prod_{i=2}^{10} \frac{(i+1)}{i} = \frac{3}{2}\frac{4}{3}\frac{5}{4}\cdots\frac{10}{9}\frac{11}{10} = \frac{11}{2}$
3. $\prod_{j=1}^{3} x^j = x \cdot x^2 \cdot x^3 = x^{1+2+3} = x^6$

Problem 2: Exponents and logarithms.

1. Using the definition of raising a number to a power, explain why $a^3 \cdot a^4 = a^{3+4}$.

   a^3 is equal to multiplying 3 copies of a together, and a^4 is equal to multiplying 4 copies together, so the product of a^3 and a^4 is equal to multiplying $3 + 4 = 7$ copies of a together:

   $$a^3 \cdot a^4 = (a \cdot a \cdot a) \cdot (a \cdot a \cdot a \cdot a) = a^{3+4}.$$

2. Simplify the following:

   (a) $\log(e^2) = 2$

   (b) $\log_2(4 \cdot 16^2) = \log_2(4) + 2\log_2(16) = 2 + 2(4) = 10$

   (c) $y^3 y^{-2} = y^{3-2} = y$

   (d) $z^{\frac{1}{2}} = \sqrt{z}$